# MATH 100 Readiness Check

Calculus will likely be one of the most challenging courses you will take in your first year university program. In order to be successful:

- You must have a *strong and recent* algebra and trigonometry background.

- You need to have a clear and realistic idea about the commitment of time and energy necessary.

The intent of this readiness check test is to help you decide whether you are ready for Math 100 or whether you should first refresh your math skills by taking the precalculus course Math 115 (Note that this course does carry University Transfer Credit and can be used for student loan purposes!).

- If you struggle with most of the problems or do not remember a large proportion of this material, you are probably not ready for Math 100 and will want to consider registering in Math 115.

- If you can work through most of the questions but struggle with some of them then you are most likely ready for Math 100. You should be prepared to devote a fair bit of time to doing math homework knowing that the quality and quantity of the time you spend will have a significant effect on your success and final grade in the course.

- If you fall somewhere between the above two situations, you will have to decide whether to stay in Math 100 or transfer to Math 115. Please talk with your instructor to help you choose the right course and strategy for you.

In your decision making process, it is important to remember that it is unrealistic to think that you can relearn most of high school algebra and trigonometry at the same time that you are learning new concepts and ideas in Calculus.

Section A: Order of operations

A.1 Evaluate the following expressions for the given values of the variables without using a calculator:

1. $\dfrac{-3x^2-2xy^3}{3x^2y^2-2y}$ if $x=-\frac{2}{3}$ $y=\frac{1}{2}$

2. $6x(8-x^2)^{\frac{4}{3}}-8x^2(8-x^2)^{\frac{1}{3}}$ if $x=-4$

3. $\dfrac{x^{-1}+y^{-1}}{x^{-1}-y^{-2}}$ if $x=4$ $y=2$

A.2 Simplify the following expressions by using an appropriate method. Give your answer using only positive exponents.

1. $5-3(2x+3)^2-(3x+1)(x-4)$

2. $\sqrt{x^2+4}+\sqrt{9x^2+36}$

3. $\dfrac{1}{x+3+h}-\dfrac{1}{x+3}$

4. $(x^{-1}-2^{-1})^{-2}$

Section B: Factoring and its Applications

B.1 Factor completely:

1. $6x^5(4x^2-1)^5+40x^7(4x^2-1)^4$

2. $2x^2+7x-15$

3. $4x^3+2x^2y-4xy^2-2y^3$

B.2 Use an appropriate technique to simplify the following expressions:

1. $\dfrac{3x^4-48}{x^2+4x-12}$

2. $\dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$

3. $\dfrac{2x}{(x-1)^2}-\dfrac{4}{x^2-x}$

B.3 Use factoring and other appropriate techniques to find all real number solutions to the following equations. You should know and may need to use the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ for some of the problems.

1. $x^4-5x^2-36=0$

2. $2x^2-3x+5=0$

3. $\dfrac{2x}{x-2}-\dfrac{x^2-3}{(x-2)^2}=0$

4. $\sqrt{x^2+1}+\dfrac{x^2-3x}{\sqrt{x^2+1}}=0$

5. $6x(x^2-1)^2(x+3)^4+4(x+3)^3(x^2-1)^3=0$ (Hint: Factor out the GCF.)

Solve the following equation for the indicated variable:

6. $A=2lw+2lh+2hw$ for $w$

7. $x=\dfrac{5-y}{y+4}$ for $y$

B.4 Solve the following inequalities and give your answer using interval notation:

1. $3-5(2x+3)+7x<0$

2. $\dfrac{3-x}{x+1}\geq 0$

## Section C: Analytic Geometry and Basic Graphing

### C.1 The Distance, Midpoint, Slope and Point-Slope Formulas

1. For the pair of points $(3,-4)$, $(-2,1)$, find:

a) The distance between the points. Recall: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

b) The midpoint of the line segment joining the points. Recall that the midpoint is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

c) The slope of the line going through the points. Recall: $m=\frac{y_2-y_1}{x_2-x_1}$

d) The slope of any line perpendicular to the line going through the points. Recall that this slope is $-\frac{1}{m}$

e) The equation of the line passing through the points. Recall the point slope form of a line: $y-y_1=m(x-x_1)$

2. Find the *x* and *y* intercepts of $3x+2y-5=0$, and then use these to sketch its graph.

### C.2 Quadratic, Rational and Radical Functions

1. For the parabola $y=-2x^2-16x-24$:

a) Find the *x* and *y* intercepts.
b) Find the vertex.
c) Sketch the graph.

2. For the rational function $y=\frac{2x-1}{x+1}$:

a) Find the *x* and *y* intercepts.
b) Identify the vertical and horizontal asymptotes.
c) Sketch the graph.

3. For the radical function $y=\sqrt{x+1}-2$:

a) Identify the domain.
b) Find the *x* and *y* intercepts.
c) Sketch the graph.

## Section D: Functions and Function Notation

D.1 Find the domain and range of the following functions:

1. $f(x)=-(x-1)^2+4$ 2. $f(x)=5-\sqrt{3-x}$ 3. $f(x)=\frac{3x-1}{x+2}$

D.2 Evaluate and simplify the following for the given function:

1. $f(-\tfrac{3}{4})$ , $f(x)=\frac{5-2\sqrt{x^2+1}}{x}$ 2. $f(a-3)$ , $f(x)=x^2-2x+5$

For each of the functions below, evaluate and simplify the difference quotient:

$$\frac{f(x+h)-f(x)}{h}$$

3. $f(x)=x^3+5$ 4. $f(x)=\frac{2}{4-x}$ 5. $f(x)=\sqrt{1-2x}$

D.3 For each pair of functions, evaluate: $f(x)g(x)$ , $f(g(x))$ and $g(f(x))$:

1. $f(x)=2x^2+5$ , $g(x)=\sqrt{2x-1}$

2. $f(x)=\dfrac{2}{3x-4}$ , $g(x)=8-6x$

Find functions $f(x)$ and $g(x)$ such that $k(x)=f(g(x))$ for the following:
(There are different correct answers for each question.)

3. $k(x)=\sqrt{9x^2+25}$

4. $k(x)=\dfrac{1}{\sqrt[5]{(x^3-27)^2}}$

D.4 Construct a function as needed for each of the following problems:

1. A piece of wire is 50 cm long and is cut into two pieces. The pieces are then used to form a square and a circle. If the length of the piece used to form the circle has a length of *x* cm, then write the total combined area *A* of the circle and the square as a function of *x*.
2. The surface area of a cylinder is given by $S=2\pi rh+2\pi r^2$ and the volume is given by $V=\pi r^2 h$ . If the volume of a cylinder is known to be 500 cc , then write the surface area as a function of only the radius *r* of the cylinder.
3. A triangle is formed in Quadrant I using the *x* and *y* axes and the line $2x+3y-6=0$ . A rectangle is drawn inside this triangle so that two sides lie on the axes and the top right corner of the rectangle is on the line. Write the area *A* of the rectangle as a function of the *x* coordinate of the point in the top right corner.

<u>Section E: Trigonometry</u>

<u>E.1 Basic Computations</u>

Convert the following degree measurement to radians or radian measurement to degrees:

1. $\dfrac{7\pi}{6}$

2. $3.42°$

Evaluate the following. You do not need to use a calculator for question 3.

3. $\sin\dfrac{5\pi}{3}$

4. $\sec\dfrac{3\pi}{5}$

Find the acute angle $\theta$ that satisfies the following equations. Give your answer in both radians and degrees. Do not use a calculator for question 5.

5. $\sin\theta=\dfrac{\sqrt{2}}{2}$

6. $\csc\theta=1.25$

Find all angles $\theta$ that satisfy the following equations. Give your answers in radians. Do not use a calculator for question 7.

7. $\sin\theta=-\dfrac{\sqrt{2}}{2}$

8. $\cos\theta=-0.6$

<u>E.2 Algebraic Computations and using Trig Identities</u>

You should know the following definitions and basic trig identities:

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ $\qquad\cot\theta=\dfrac{\cos\theta}{\sin\theta}$ $\qquad\sec\theta=\dfrac{1}{\cos\theta}$ $\qquad\csc\theta=\dfrac{1}{\sin\theta}$

$\sin^2\theta+\cos^2\theta=1$ $\qquad 1+\tan^2\theta=\sec^2\theta$ $\qquad 1+\cot^2\theta=\csc^2\theta$

$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$ $\qquad\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

$\sin 2x=2\sin x\cos x$ $\qquad\cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$

Note that some of the above identities are interrelated, for example:

$$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \Rightarrow \tan^2\theta + 1 = \sec^2\theta$$

$$\sin 2x = \sin(x + x) = \sin x\cos x + \cos x\sin x = 2\sin x\cos x$$

Simplify the following expressions using appropriate algebraic techniques and/or trig identities:

1. $\dfrac{\sec^2 x - \tan^2 x}{\sec^2 x - 1}$
2. $\dfrac{1-\sin x}{\cos^2 x}$
3. $\dfrac{1}{1-\cos x} - \dfrac{1}{1+\cos x}$
4. $\sin(x+y)\sin(x-y)$
5. $\cos^2 4x - \sin^2 4x$
6. $\dfrac{\sin 2x}{1+\cos 2x}$

E.3 Use trig identities to help in solving the following equations. Give your answer in radians and find all solutions on the interval $[0, 2\pi)$

1. $\sin 2x + 2\cos x = 0$
2. $\cos 2x = -\cos x$
3. $3\sec x\tan x - 2 = 0$

E.4 Find functions $f$ and $g$ so that $k(x) = f(g(x))$:

1. $k(x) = \sin x^3$
2. $k(x) = 2\cos x + 1$

Find functions $f$ , $g$ and $h$ so that $k(x) = f(g(x))$

3. $k(x) = 3\cos x^4 + \dfrac{5}{\cos x^4}$

E.5 Sketch the graphs of the following functions on the interval $[0, 2\pi]$:

1. $y = \cos 2x$
2. $y = \sin x - 1$
3. $y = \tan\left(\dfrac{x}{2}\right)$
4. $y = 3\cos\frac{1}{2}(x+\pi)$

# ANSWERS

Section A.1

1. $\frac{7}{4}$ 2. $-128$

3. Recall that $x^{-1} = \frac{1}{x}$ and $y^{-2} = \frac{1}{y^2}$. The resulting expression is undefined because division by zero is not defined.

Section A.2

1. $-15x^2 - 25x - 18$ 2. $4\sqrt{x^2 + 4}$

3. $\frac{-h}{(x+3+h)(x+3)}$ 4. $\frac{4x^2}{(2-x)^2}$ or $\frac{4x^2}{(x-2)^2}$

Section B.1

1. $2(x^2+1)^2(5x+1)(20x^2+3x+5)$ 2. $(2x-3)(x+5)$
3. $2(2x+y)(x-y)(x+y)$

Section B.2

1. $\frac{3(x+2)(x^2+4)}{x+6}$ 2. $-\frac{1}{2x}$ 3. $\frac{2(x^2-2x+2)}{x(x-1)^2}$

Section B.3

1. $x = \pm 3$ 2. no real solutions
3. $x = 1$, $x = 3$ 4. $x = \frac{1}{2}$, $x = 1$
5. Factoring gives: $2(x+3)^3(x^2-1)^2(5x^2+9x-2) = 0$. Factoring further gives:
$2(x+3)^3(x-1)^2(x+1)^2(5x-1)(x+2) = 0 \Rightarrow x = -3, x = 1, x = -1, x = -2, x = \frac{1}{5}$
6. $w = \frac{A - 2lh}{2l + 2h}$ 7. $y = \frac{5-4x}{x+1}$

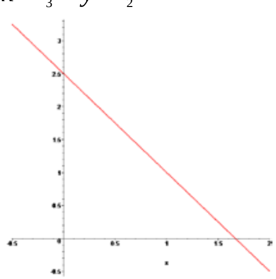
Section B.4

1. $x > -4$ or $(-4, \infty)$ 2. $(-1, 3]$

Section C.1

1. a) $5\sqrt{2}$ b) $\left(\frac{1}{2}, -\frac{3}{2}\right)$ c) $-1$ d) 1 e) $y = -x - 1$ or $x + y + 1 = 0$

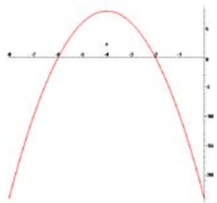
2. $x = \frac{5}{3}$ $y = \frac{5}{2}$



Section C.2

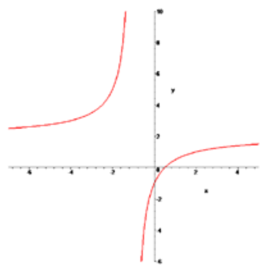
1. a) $x=-2$ $x=-6$ and $y=-24$ b) $(-4\,,8)$

   c)



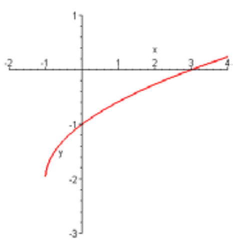
2. a) $x=\frac{1}{2}$ $y=-1$ b) $x=-1$ $y=2$

   c)



3. a) $x\geq -1$ b) $(3,0)$ and $(0,-1)$

   c)



Section D.1

1. The graph is an upward opening parabola with vertex ( -3, 2 ) so:
   Domain: all real numbers Range: $y\geq 2$
2. Domain: all real numbers Range: $y\leq 4$
3. Domain: $2x+1\geq 0 \Rightarrow x\geq -\frac{1}{2}$

   Range: Since $\sqrt{2x+1}$ is greater than or equal to zero, the range is $y\geq -2$
4. Domain: $x\leq 3$ Range: $y\leq 5$
5. Domain: $5x-4\neq 0 \Rightarrow x\neq \frac{4}{5}$ or $\{\, x \mid x\neq \frac{4}{5}\,, x$ a real number $\}$

   Range: $\dfrac{3}{5x-4}$ can not be zero so $y\neq 1$ or $\{\, y \mid y\neq 1\,, y$ a real number $\}$
6. Use long division to rewrite the function as $f(x)=3-\dfrac{7}{x+2}$.

   Domain: $x\neq -2$ Range: $y\neq 3$

Section D.2

1. $-\frac{10}{3}$

2. $f(a-3)=a^2-8a+20$

3. $3x^2+3xh+h^2$

4. $\dfrac{2}{(4-x-h)(4-x)}$

5. $\dfrac{-2}{\sqrt{1-2x-2h}+\sqrt{1-2x}}$

Section D.3

1. $f(x)g(x) = (2x^2+5)\sqrt{2x-1}$ ; $f(g(x)) = 4x+3$ , $x \geq \frac{1}{2}$ ; $g(f(x)) = \sqrt{4x^2+9}$
2. $f(x)g(x) = -4$ , $f(g(x)) = \dfrac{1}{10-9x}$ , $g(f(x) = \dfrac{24x-44}{3x-4}$
3. $f(x) = \sqrt{x}$ , $g(x) = 9x^2+25$
4. $f(x) = x^{-2/5}$ , $g(x) = x^3 - 27$

Section D.4

1. Radius of the circle is $r = \dfrac{x}{2\pi}$ and the length of a side of the square is $s = \dfrac{50-x}{4}$.

   So, $A(x) = \pi\left(\dfrac{x}{2\pi}\right)^2 + \left(\dfrac{50-x}{4}\right)^2 = \dfrac{x^2}{4\pi} + \dfrac{x^2-100x+2500}{16}$

2. $500 = \pi r^2 h \Rightarrow h = \dfrac{500}{\pi r^2}$

   Therefore: $S(r) = 2\pi r\left(\dfrac{500}{\pi r^2}\right) + 2\pi r^2 \Rightarrow S(r) = \dfrac{1000}{r} + 2\pi r^2$

3. $A(x) = 2x - \frac{2}{3}x^2$

Section E.1

1. $210°$
2. 0.06 radians
3. $-\dfrac{\sqrt{3}}{2} = -0.866$
4. -3.236
5. $45° = \dfrac{\pi}{4}$ rads
6. $53.13° = 0.927$ rads
7. $\theta = \dfrac{5\pi}{4} + 2\pi k$ , $\theta = \dfrac{7\pi}{4} + 2\pi k$
8. $\theta = 2.214 + 2\pi k$ , $\theta = 4.069 + 2\pi k$

Section E.2

1. $\dfrac{1}{\tan^2 x} = \cot^2 x$

2. $\dfrac{1}{1+\sin x}$

3. $\dfrac{2\cos x}{\sin^2 x} = 2\csc x \cot x$

4. $(\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y) = \sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

   $= \sin^2 x - \sin^2 y$ or alternatively $= \cos^2 y - \cos^2 x$

5. $\cos 8x$

6. $\dfrac{2\sin x\cos x}{1+(2\cos^2 x-1)} = \dfrac{2\sin x\cos x}{2\cos^2 x} = \tan x$

Section E.3

1. $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$

2. $\{\frac{\pi}{3}, \frac{5\pi}{3}, \pi\}$

3. $3\left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right) = 2 \Rightarrow 3\sin x = 2\cos^2 x \Rightarrow 3\sin x = 2(1-\sin^2 x)$

$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \Rightarrow (2\sin x - 1)(\sin x + 2) = 0 \Rightarrow x = \frac{\pi}{6} \; or \; x = \frac{5\pi}{6}$

Section E.4

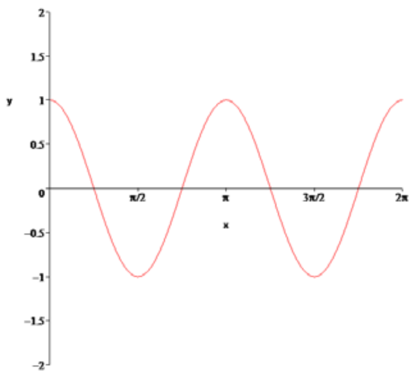
1. $f(x) = \sin x \quad g(x) = x^3$

2. $f(x) = 2x+1 \quad g(x) = \cos x$

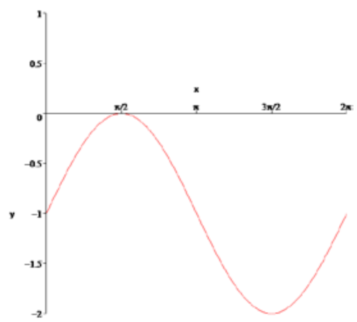
3. $f(x) = 3x + \frac{5}{x} \quad g(x) = \cos x \quad h(x) = x^4$

Section E.5

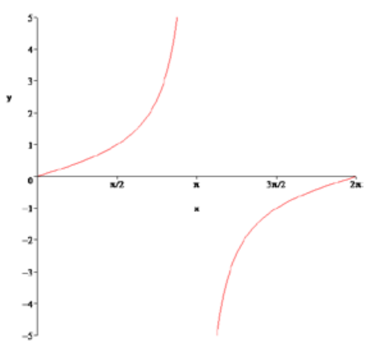
1. Amplitude =1 Period = $\frac{2\pi}{2} = \pi$




2. Vertical shift down 1




3. Period = $\frac{\pi}{½} = 2\pi$




4. Amplitude = 3 Period = $\frac{2\pi}{½} = 4\pi$ Phase Shift = $\pi$ to the left